fi360 Asset Allocation Optimizer: Risk-Return Estimates*

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This report provides risk-return estimates for twelve asset classes for use with the fi360 Asset Allocation Optimizer. The risks and returns are computed from long-term historical data overlaid with financial theory and adjusted for current interest rates. The returns represent "equilibrium" market estimates relative to the selected optimization universe adjusted for short-term market rates for contemporary realism. A market equilibrium approach is designed to provide a longer term perspective on capital market rates by limiting period-dependence as well as avoiding ad hoc "building block" methods in current usage. The estimates are useful to illustrate properties of optimized portfolios for Markowitz mean-variance (MV) and Michaud and Michaud (2008, Ch. 6) Resampled Efficient Frontier™ (REF) methodologies. While also useful as a framework for comparative purposes relative to a given set of forecasts, judgment is required for application to an investment program.[1]

Indices and Historical Return Data

Historical returns of major stock and bond capital market indices are the starting points for estimating asset risks and returns for asset allocation studies. Reliable index returns are available from 1926 for a few major asset classes.[2]

Column 1 in Table 1 lists the twelve asset classes in the fi360 asset allocation demonstration system. Column 2 in Table 1 lists the actual index data used as the proxy for the indicated asset class.[3] Column 3 displays the starting point of the series of monthly returns for each index. The ending point for the monthly data used in the study is December 2012.

Monthly returns are statistically convenient for reliable estimation of correlations. The choice of representative index may often require some compromise of longevity versus relevance and representativeness for the asset class.

Major U.S. Indices and Return Premiums

Reliable major U.S. capital market monthly historical stock and bond index returns have been available since 1926.[4] Inflation and short-term interest rates vary widely over a long-term historical period. To improve estimate relevance it is generally appropriate to compute historical risk-return estimates relative to inflation or short-term interest rates. These real risk-return estimates can then be redefined in current nominal terms by the addition of current interest rates.

[1] In practice markets are not in equilibrium. More fundamentally, any set of risk-return estimates are generally investment useful primarily for a targeted group of investors. Risk-return estimates vary by institution and by investor considerations such as objectives, horizons, funds for investment, and risk aversion.

[2] Ibbotson (2011).

[3] 2011 and 2012 index returns are taken from Morningstar Principia for the corresponding Ibbotson indices.

[4] The Center for Research in Security Prices (CRSP) stock return databases began in 1926. Original studies of CRSP stock market returns and risk include Fisher and Lorie (1968, 1970).

Table 1
Asset Classes, Indices, Range of Monthly Returns

Asset Class	Index	Beginning Returns
Fixed Income	SBBI 30 day US Treasury Bill	Jan-26
Intermediate Term Bond	Barcap U.S. Interm. Gov/Credit Bond	Jan-73
Long Term Bond	Barcap U.S. Long Gov/Credit Bond	Jan-73
High Yield Bonds	Credit Suisse High Yield Bond Index	Jan-86
International Govt Bonds	Citigroup World Gov Bond ex US	Feb-85
Commodities	Gold, London PM Fix	Jan-74
Large Cap Equity	SBBI Large Company Stocks	Jan-26
Mid Cap Equity	Russell Mid Cap	Feb-79
Small Cap Equity	SBBI Small Company Stocks	Jan-26
International Equity	MSCI EAFE	Jan-70
Emerging Market Equity	MSCI Emerging Markets	Jan-88
REITs	DJ US Select REIT	Jan-78

The average annualized monthly return of large capitalization U.S. stocks from 1926 through 2012 is 11.2% with a standard deviation of 19.1%. T-bill returns averaged 3.5% for the period with a 0.9% standard deviation. The large cap and T-bill return premium relative to inflation (CPI) for the period is 8.2% and 0.5% respectively with 19.2% and 1.8% standard deviations. T-bills have a 0.08 CPI relative correlation with large cap equity indices over this time period.[5]

Relevant Index Return

Asset risk-return estimation requires addressing the relevance of historical return data. While long term historical returns are available for a few asset classes from the early and mid-twentieth century, it is of interest to consider whether it is very relevant for investment purposes in the early twenty-first century. Revolutionary developments have occurred in communications, derivatives, regulation, accounting, politics, investment strategies, and economics in the last ten to twenty years. Many international and fixed income indices have no reliable return data prior to 1970. In addition, the gold standard – in effect prior to 1973 – seriously compromises the estimation of correlations among many asset classes. On the other hand, statistical reliability, particularly with respect to correlation estimation and return premiums require as much long-term data as is reasonable.

To satisfy these competing priorities we begin our estimation period with the availability of reliable international equity data unaffected by the gold standard. We use January 1973 as the starting point for our data set. As Table 1 indicates, many of the indices in the study have missing data over the 1973 to 2012 period. The most affected are high yield bonds, international government bonds, and emerging equity markets. Our study uses a sophisticated maximum likelihood statistical procedure designed to consistently

[5] Estimates are Ibbotson stocks, bonds, bills and inflation. *Ibbotson SBBI 2011 Classic Yearbook*, Chicago, Il. 2011 and 2012 returns are also taken from Morningstar Principia.

and holistically estimate risk-return parameters in the presence of missing data.[6] This missing data algorithm avoids arbitrary decisions for parameter estimation common in many prior asset allocation studies.[7]

Geometric versus Arithmetic Returns

One of the more enduring misunderstandings of estimating risks and returns for asset allocation optimization is the confusion between geometric, or compound, and arithmetic returns. A familiar example demonstrates the difference between the arithmetic and geometric mean of returns. Assume a 100% return followed by a -50% return. The arithmetic mean return over the two periods is 25% but a dollar grows to 2 dollars and ends up at 1 dollar at the end of the second period. The geometric mean gives the correct measure of return over the two periods, 0%.

Which return measure, arithmetic or geometric, is appropriate as a basis for defining risk and return for portfolio optimization? As Michaud (1981, 2003) shows, the geometric mean is a function of the length of the investment horizon as well as the return distribution. In contrast, the arithmetic mean is a measure of centrality that does not depend on the investment horizon. The arithmetic mean is the measure of choice for portfolio optimization. From a practical point of view, the geometric mean typically underestimates investment return and may lead to unrealistic risk-averse investment decisions.

Index Choice and Risk-Returns

The long-term U.S. indices are from Ibbotson. Intermediate and Long Term Bonds are Barclays Capital U.S. Intermediate and Long Government Credit Bonds respectively[8]. The Russell Midcap index is available from 1979.[9] The Morgan Stanley Capital International (MSCI) Europe Asia and Far East (EAFE) and Emerging Market indices are used for defining risk-return estimates for non-U.S. stocks.[10] These are standards for international investment, as extensively available as any other set of indices, and managed relative to float, investability and other issues of concern in foreign markets. The CSFB Credit Suisse High Yield and Citigroup WGBI non-U.S. Government Bond index are relevant long-lived bond indices.[11] The Dow Jones US Select REIT[12] index is a relevant long-lived REIT index. The London, PM Fix gold index may usefully represent

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[6] A standard reference is Carlin and Louis (1992). The investment problem raised with any missing data estimation procedure is that the existence of an index would have modified the investment environment and may have affected other index returns.

[7] See the discussion on the use of the EM algorithm in Michaud et al (2006).

[8] Morningstar Principia © 2012 Morningstar Inc. Barclays Capital U.S. Gov. Credit Bond Indices are formerly Lehman Brothers US Gov. Credit Bond Indices.

[9] Russell Investment Group. "U.S. Equity Index Values." *Russell.* www.russell.com/indexes/performance/calculator/calculator.asp (January 24, 2012).

[10] Morgan Stanley Capital International. "Equity Indices." *MSCI.* http://www.mscibarra.com/products/indices/international_equity_indices/performance.html (January 24, 2012).

[11] Morningstar Principia © 2011 Morningstar Inc.

[12] The DJ Wilshire REIT index is now known as The Dow Jones US Select REIT index.

commodities.[13] The MSCI Emerging Markets index, available since January 1988, is the shortest period index. The 1973-2012 historical period provides sufficient historical return data for risk-return estimation. Table 2 summarizes our historical CPI adjusted risk and return estimates.

Table 2
Annualized CPI Relative Risk-Return Estimates: 1973-2012

Asset Classes	Return (%)	Std. Deviation (%)
Fixed Income	1.0	1.2
Intermediate Term Bond	3.3	4.6
Long Term Bond	4.9	10.1
High Yield Bonds	6.4	8.5
International Govt Bonds	5.1	10.6
Commodities	4.7	19.8
Large Cap Equity	6.4	15.8
Mid Cap Equity	8.8	17.9
Small Cap Equity	10.6	21.8
International Equity	5.6	17.8
Emerging Market Equity	12.9	24.4
REITs	9.3	19.2

Equilibrium Return Estimation

Historical return estimates typically reflect time-period dependent results that are unlikely to be useful for long-term investment objectives. For example, Table 2 reports that international equity has a lower estimated return than REITs, a result that many investors will think unlikely to persist over any significant future time period. Various ad hoc methods such as additive "building blocks" based on historical market premiums have been proposed to reduce period-dependent results in risk-return estimation. Our estimates are based on fundamental principles of capital market functioning. The use of the Security Market Line (SML) framework of the Capital Asset Pricing Model (CAPM) provides a financially relevant framework for limiting near-term period-dependent results.

We estimate SML returns from beta relationships relative to the market portfolio. The procedure uses linear regression of historical asset returns relative to a capitalization weighted global equity market proxy for CPI relative data.[14] CAPM SML return estimates are calculated from their estimated betas and the 5.4% return premium from Table 2. The regression results are based on available CPI adjusted return data for each asset over the 1973-2012 estimation period.

[13] See Michaud, Michaud, and Pulvermacher (2006) for more discussion of commodity indices generally and gold in particular.
[14] Capitalization weights are from MSCI.

Table 3 provides risk estimates from Table 2 and CAPM SML return estimates as calculated from their estimated betas relative to the 5.4% (large cap minus fixed income) return premium. The correlations used in the optimization process are based on the historical return data for the 1973-2012 period underlying Table 2 and are included in the Appendix. Ledoit estimation (Ledoit and Wolf 2003), a standard methodology for addressing estimation error in correlations, is applied to the correlation estimates.

Our estimate of commodity return in Table 2 reflects a time-period dependent boom that is not consistent with long term market equilibrium. From first principles, the long term return for a commodity should be the inflation rate. Table 3 includes an estimate for commodities equal to the inflation rate or zero real return that is consistent with an equilibrium framework. Table 4 summarizes our nominal risk-return results using a 2% inflation rate where commodities are set to inflation.[15]

Table 3
CAPM SML CPI Adjusted Risk-Return Estimates

Asset Classes	Return (%)	Risk (%)
Fixed Income	1.0	1.2
Intermediate Term Bond	1.3	4.6
Long Term Bond	1.8	10.1
High Yield Bonds	2.7	8.6
International Govt Bonds	1.9	10.6
Commodities	0	19.8
Large Cap Equity	6.0	15.8
Mid Cap Equity	6.4	17.9
Small Cap Equity	6.7	21.8
International Equity	6.6	17.8
Emerging Market Equity	7.3	24.4
REITs	5.0	19.2

Summary

The purpose of this study is to provide risk-return estimates for twelve major capital market asset classes for use in the fi360 Asset Allocation Optimizer. Historical monthly return data for the 1973-2012 periods was used as the basis of risk-return estimates. Adjustments for the CPI, commodities, current inflation rate assumptions, missing date technology, and CAPM equilibrium methodology led to estimates of return. Risk relationships were estimated from the historical data and the EM algorithm to account for missing data in some of the twelve indices. The final risk-return estimates are given in Table 4; the correlations for input to the optimizer are given in the appendix.

[15] See, e.g., http://www.forecasts.org/inflation.htm and http://www.forecast-chart.com/forecast-inflation-rate.html

Table 4
Nominal CAPM Risk-Return Estimates
2% Inflation Assumption: 1973-2012 Historical Returns

Asset Classes	Return (%)	Risk (%)
Fixed Income	3.0	1.2
Intermediate Term Bond	3.3	4.6
Long Term Bond	3.8	10.1
High Yield Bonds	4.7	8.5
International Govt Bonds	3.9	10.6
Commodities	2.0	19.8
Large Cap Equity	8.0	15.8
Mid Cap Equity	8.4	17.9
Small Cap Equity	8.7	21.8
International Equity	8.6	17.8
Emerging Market Equity	9.3	24.4
REITs	7.0	19.2

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Appendix

Historical Correlations, Benchmark Relative to CPI: 73-12, Ledoit Estimated

Money Market	Intermediate Term Bond	Long Term Bond	High Yield Bonds	International Govt Bonds	Commodities	Large Cap Equity	Mid Cap Equity	Small Cap Equity	International Equity	Emerging Market Equity	REITs
1.00	0.42	0.31	0.10	0.18	-0.13	0.15	0.11	0.07	0.13	0.04	0.06
0.42	1.00	0.88	0.33	0.51	0.06	0.23	0.22	0.12	0.19	0.09	0.19
0.31	0.88	1.00	0.35	0.43	0.04	0.27	0.26	0.15	0.19	0.10	0.22
0.10	0.33	0.35	1.00	0.14	0.06	0.61	0.68	0.66	0.51	0.55	0.61
0.18	0.51	0.43	0.14	1.00	0.30	0.08	0.06	-0.01	0.45	0.13	0.11
-0.13	0.06	0.04	0.06	0.30	1.00	0.02	0.06	0.04	0.19	0.24	0.06
0.15	0.23	0.27	0.61	0.08	0.02	1.00	0.93	0.76	0.64	0.67	0.60
0.11	0.22	0.26	0.68	0.06	0.06	0.93	1.00	0.88	0.63	0.70	0.69
0.07	0.12	0.15	0.66	-0.01	0.04	0.76	0.88	1.00	0.53	0.66	0.67
0.13	0.19	0.19	0.51	0.45	0.19	0.64	0.63	0.53	1.00	0.67	0.47
0.04	0.09	0.10	0.55	0.13	0.24	0.67	0.70	0.66	0.67	1.00	0.49
0.06	0.19	0.22	0.61	0.11	0.06	0.60	0.69	0.67	0.47	0.49	1.00